1 OBJECTIVES
Long after Turing’s seminal Reaction-Diffusion (RD) model, the elegance of his fundamental equations alleviated much of the skepticism surrounding pattern formation. Interestingly, we serve Turing-like patterns in a system of neurons with adversarial interaction. In this study, we establish the following:

1. Involvement of Turing instability.
2. A Pseudo-Reaction-Diffusion model.

2 INTRODUCTION
In this paper, we intend to demystify an interesting phenomenon: adversarial interaction between generator and discriminator creates non-homogeneous equilibrium by inducing Turing instability in a Pseudo-Reaction-Diffusion (PRD) model. This is in stark contrast to sole supervision. Thus we state our key observation:

A system in which a generator and a discriminator adversarially interact with each other exhibits Turing-like patterns in the hidden layer and top layer of the two layer generator network.

3 PRELIMINARIES

3.1 Supervised Training:
\[ L_{adv}(U) = \frac{1}{2} \sum_{j} \| V_j(U) - V_j \|_2 \]

3.2 Regularized Adversarial Training:
\[ L_{adv}(U, V) = \frac{1}{2} \sum_{j} \| V_j(U) - V_j \|_2 \]

Learning Algorithm:
\[ \frac{dU_{jk}}{dt} = -\frac{\partial L_{adv}(U(t), V(t), W(t), a(t))}{\partial U_{jk}(t)} \]
\[ \frac{dV_{jk}}{dt} = -\frac{\partial L_{adv}(U(t), V(t), W(t), a(t))}{\partial V_{jk}(t)} \]

Pseudo-Reaction-Diffusion Model [1]:
\[ \frac{du_{jk}}{dt} = \nabla^2 u_{jk} + \mathcal{D}^u (\nabla^2 u_{jk}) \]
\[ \frac{dv_{jk}}{dt} = \nabla^2 v_{jk} + \mathcal{D}^v (\nabla^2 v_{jk}) \]

4 THEORETICAL ANALYSIS

(Informal) Theorem 1. (Symmetry and Homogeneity) Suppose the necessary assumptions hold. We obtain the following with probability at least 1 - \( \delta \):
\[ \| u_j(t) - u_j(0) \|_2 \leq C \left( \frac{n^{3/2}}{\lambda} \left( 1 - \exp \left( -\frac{\lambda t}{2} \right) \right) \right) \]

(Informal) Theorem 2. (Breakdown of Symmetry and Homogeneity) If the required conditions are satisfied, then with probability at least 1 - \( \delta \), we get
\[ \| u_j(t) - u_j(0) \|_2 \leq \mathcal{D} \left( \frac{n^2}{\lambda^2} \left( 1 - \exp \left( -\frac{\lambda t}{2} \right) \right) \right) + \left( \frac{\rho(1 + \sqrt{\rho})}{\lambda} \right) \]

Analogous Bernoulli Differential Equation: Modeling Population Growth,
\[ \frac{dP}{dt} = rP \left( 1 - \frac{P}{K} \right) \]

Modeling Regularized Adversarial Training,
\[ \frac{d\psi}{dt} \leq r^{1/2} \left( 1 - \frac{\psi^{1/2}}{K} \right) \]

5 EXPERIMENTAL RESULTS

Figure 1: Distance from multiple initialization in the (a) hidden layer and (b) top layer on MNIST.

Figure 2: Input image used for the visualization of features in the hidden layer.

Figure 3: Hidden layer filters on MNIST. (a) Without Diffusion. (b) With Diffusion.

Figure 4: Visualization of features on MNIST. (a) Without Diffusion. (b) With Diffusion.

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Figure 5: Breakdown of symmetry and homogeneity. (a) Without Diffusion. (b) With Diffusion.

Figure 6: Turing pattern formation. The diffusible factors help break the symmetry and homogeneity.

Figure 7: Pattern formation on synthetic data, \( d_{in} = 784 \) without Diffusion.

Figure 8: Pattern formation on synthetic data, \( d_{in} = 784 \) with Diffusion.

7 FUTURE SCOPE
Though diffusibility ensures more local interaction, it will certainly be interesting to synchronize neurons based on breakdown of symmetry and homogeneity in the future.

REFERENCE


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