Solving Linear Inverse Problems Provably via Posterior Sampling using Latent Diffusion Models

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Source Code: github.com/LituRout/PSLD

Introduction to Inverse Problems

- Groundtruth Image $x_0 \sim p_{data}(x)$
- Imaging System $A$
- Noise $\eta$
- Measurements $y = A(x_0) + \eta$

Problem: Reconstruct groundtruth image $x_0$ from noisy measurements $y$
Challenge: Problem is ill-posed, that is infinitely many solutions $x_0$ exist
Approach: Use prior knowledge $p(x_0)$ of how the image should look like

Background on Diffusion Models

- Diffusion models have emerged as powerful priors for inverse problems!
- Forward SDE: $dx_t = f(x_t, t)dt + g(t)dw_t$
- Reverse SDE: $dz_t = (z_t + 2\nu \log p_t(z_t))dt + \sqrt{2}\nu dw_t$

Posterior Sampling using Diffusion Models

- Conditional SDE: $dz_t = (z_t + 2\nu \log p_t(z_t))dt + \sqrt{2}\nu dw_t$

Problem: How well can we approximate $\nu \log p_t(z_t)$?

Posterior Sampling using Latent Diffusion Models (Our Approach)

- Problem: Need $p_{true}(y|z)$, what we can compute instead is $p_{true}(y|z)$

DPS (Chung et al., KIFC23):

- $p_{true}(y|z) = p_{true}(y|x) p_{true}(x|z) = p_{true}(y|x) p_{true}(z)$
- can be computed using Tweedie’s formula

Experimental Results: Real-world Images

(a) Input
(b) Groundtruth
(c) Comm. Serv. 1
(d) Comm. Serv. 2
(e) PSLD (Ours)

Experimental Results: Motion Deblur (left) and Box Inpainting (right)

Experimental Results: Web Application

(input) Groundtruth (DPS) PSLD (Ours) (input) Groundtruth (DPS) PSLD (Ours)

PSLD (Ours) (Proposed): Denote by $\tilde{z}_T = Dec(Enc(z_T))$. Then:

- VAE is trained using $Enc(Enc(x)) = z_0$, $z_0 \sim \mathcal{N}(0,I)$
- Ideally, $x_T = Dec(z_T)$ is a natural image and $Enc(Dec(z_T)) = z_T$
- In practice, $Enc(Dec(z_T)) \approx z_T$, which causes instability and inconsistency
- Stability: Look for $z_T$ satisfying $Enc(Dec(z_T)) = z_T$
- Consistency: Make sure that $A^T y + (I - A^T A) Dec(z_T)$ is a natural image