Optimal Transport in Generative Models

**Monge Problem (MP):**
- Cost $\psi(b)$ Pushforward
- The feasible set of solutions can be empty.
- MP does not allow mass splitting.

**Kantorovich Problem (KP):**
- Cost $\psi(b)$ Pushforward
- Kantorovich Dual Problem (KDP):

**Optimal Transport as the Generative Map**

- Compute OT maps in latent spaces of autoencoders.
- OT maps are not considered in ambient spaces.
- Lack scalability due to poor expressivity of ICNNs.

**Proposed Optimal Transport Modeling**

### Equal Dimensions of Input and Output

With $\psi(y) \triangleq \|y\|^2 - \|x\|^2$, the $\psi$-transform of Kantorovich potential $\psi(y)$ becomes:

$$v^\ast(x) = \inf_{\mu \in \mathcal{P}, \nu \in \mathcal{Q}} \int x \, d\mu + \int y \, d\nu - \frac{1}{2} x^2 - \sup_{\psi \in \mathcal{P}} \int \psi(x, y) - \psi(y) \, d\nu.$$

### Unequal Dimensions of Input and Output

$$\sup_{\psi \in \mathcal{P}} \int \psi(x, y) - \psi(y) \, d\nu$$

- We prove that the OT map $T^*$ is the solution of the inner maximization problem (2), see Lemma 4.2 in our paper.